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Optimal Cruciform Specimen Design Using the Direct Multi-search Method and Design Variable Influence Study

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Abstract

Nowadays the development of new testing machines and the optimization of new specimen geometries are two very demanding activities. In order to study complex material stress and strain distributions, as in-plane biaxial loading, one must develop new technical solutions. A new type of testing machine has been developed by the present authors, for the fatigue testing of cruciform specimens, but the low capacity of the testing machine requires the optimization of the specimen in order to achieve higher but uniform stress and strain distributions on the specimen center. In this paper, the authors describe the procedure to optimize one possible geometry for cruciform specimens, able to determine the fatigue initiation life of material subjected to out of phase in-plane biaxial fatigue loadings. The high number of design variables were optimized using the direct multi-search method, considering two objective functions, the stress level on the specimen center and the uniformity of the strain distribution on a 1.0 mm radius of the specimen center. Several Pareto Fronts were obtained for different material thickness, considering the commercially available sheet metal thickness. With the optimal solution, the influence of every design variable was studied in order to provide others with a powerful tool that allows selecting the optimal geometry for the desired application. The results are presented in the form of design equations considering that the main design variable, the material thickness, was chosen from a Renard series of preferred numbers. The end user is then able to configure the optimal specimen for the required fatigue test.

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Keywords: Optimization; Cruciform Specimen; Fatigue Initiation Life; Pareto Fronts; Renard Series.

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1. Introduction

With the introduction of new test machines, based on full electrical linear motors, development of new test specimens is required. These test machines have amazing features when compared with the traditional hydraulic systems, like the cost, almost none maintenance, higher dynamic capacity and much more energy efficient; however, the load capacity is much above. One of such examples is the biaxial in-plane test machine built with four iron core linear motors, developed by the authors Freitas et al (2013). This machine can apply complex biaxial loading paths to most of the engineering materials but the maximum force capacity of each motor in dynamic conditions is about ±3.5 kN. This requires that part of the traditional cruciform specimen geometries must be redesigned and optimized in order to become appropriated for these test machines capacity.

Along of the years several cruciform geometries have been presented as summarized by Kuwabara (2007). Most of the specimens present design improvements like the radius of the corner fillet at the interception of the arms, the shape and the thickness of the center, the width of the arms and some create strip and slots at the arms in order to increase the area with uniform stress on the center. In this work the authors are interested in to optimize a cruciform geometry appropriated for fatigue crack initiation. Most of the cruciform geometries found in literature are not appropriate for fatigue testing and in special for crack initiation. The main reason is that stress concentrations, that are promoted by some features, makes that failure may occur outside the center of the specimen, invalidating the fatigue test.

From all geometries found in the literature, the one proposed by Wilson and White (1971), seems the most promising for fatigue crack initiation. This geometry consists of a reduced center thickness, with minimal stress concentration, and an appropriate curvature in the transition between the center of the specimen to the arms to avoid stress concentrations. In this paper this cruciform geometry is optimized for low loads, maximizing the central thickness as large as possible, maintaining the strain level in the center uniform (in certain limits) and ensuring that failure will occur in the center. As optimizer it was used the Direct Multi-Search (DMS) methodology to obtain several Pareto Fronts relating the objectives functions. The results of the optimized geometry were organized in such way that an end user can design his own specimen regarding their needs and using the equations provided.

Nomenclature

DMS	Direct Multi-Search
dd	elliptical fillet center position
h and k	spline definition parameters
R_m	major elliptical fillet radius
r_m	minor elliptical fillet radius
rr	revolved center spline major radius
t	specimen arms thickness
tt	specimen center reduced thickness
θ	revolved center spline exit angle

2. Multi-objective optimization

In this paper a multi-objective nonlinear programming problem approach proposed by Custodio et al (2012), Franco Correira et al (2016), Miettinem (1999) or Kalyanmoy Deb (2001) is considered:

Find n design variables $\boldsymbol{x} = (x_1, x_2, \dots, x_n)^T \in \Omega \subseteq \mathbb{R}^n$
which minimize:

$$min_{x \in \Omega} F(\boldsymbol{x}) \equiv \left(f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \dots f_k(\boldsymbol{x})\right)^T \tag{1}$$

involving $k > 1$ objective functions or objective function components $f_i \colon \Omega \subseteq \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}$, $i = 1, \dots, k$ and being Ω a feasible region. Furthermore, it is assumed that all the objective functions to be of the black-box type, meaning that only function values are available and can be used to solve the problem. This is a common feature in many situations in which computation of the problem functions is the result of time-consuming and complex simulation programs.

Multi-objective constrained problems, the situation in which two (or even more) conflicting performances are to be optimized is becoming more and more frequent in practice, and, more particularly, black-box type, are almost ubiquitous in real-world applications and very well studied in the literature Custodio et al (2012), Franco Correira et al (2016), Araújo et al (2013), Madeira et al (2015). In the presence of several objective functions, the set of design variables, which minimize one function, are not necessarily the same, which minimize another function. In such situations, the classical optimality definition for single-objective problems must be replaced by the well-known Pareto optimality definition.

DMS Custodio et al (2012) is a derivative-free method for multi-objective optimization problems, which does not aggregate any components of the objective function. It is inspired by the search/poll paradigm of direct-search methods of directional type, extended from single to multi-objective optimization and uses the concept of Pareto dominance to maintain a list of feasible non-dominated points. The new feasible points evaluated in each iteration are added to this list and the dominated ones are removed. Successful iterations correspond then to changes in the list, meaning that a new feasible non-dominated point was found. Otherwise, the iteration is declared as unsuccessful.

In the DMS method, the constraints are handled using an extreme barrier function. When a point is unfeasible, the components of the objective function F are not evaluated, and the values of $\boldsymbol{F}_{\Omega}$ are set to $+\infty$. This approach allows dealing with black-box type constraints, where only a yes/no type of answer is returned. Details are omitted in the present paper and the reader is referred to Custodio et al (2012) for a more complete description of this method.

3. Materials and Methods

3.1. Specimen Geometry

The goal of the paper is to optimize the geometry of cruciform specimens. When subjected to biaxial loads, these specimens present high stress levels on the arms corner and therefore a solution for this problem must be developed. Based on previous experience the authors used an elliptical fillet in order to reduce the stress concentration on the specimen arms, as seen in Fig. 1. Unfortunately, this design detail is not enough to make sure that the higher stress level always occurs on the specimen center. A center revolved spline was then used to reduce the specimen center thickness, Fig. 1 and 2, to always achieve the higher stress levels on the specimen center, making sure that the initial fatigue crack always happens on the specimen center.

The revolve spline is an optimal feature to achieve this goal. It is tangent to the horizontal direction at the specimen center; therefore, the lowest thickness is always present in this point and consequently the maximum stress . The spline also allows to control the uniformity of the strain distribution on the specimen center, by controlling the position and the inclination of the oblique line, Fig. 2, that is tangent to the spline. Finally, the exit angle of the spline controls the stress distribution around the revolve area, increasing the difference between the center stress and arms stress levels. As seen in previous works, this geometry allows for high and uniform stress levels on the specimen center, appropriated for fatigue crack initiation.

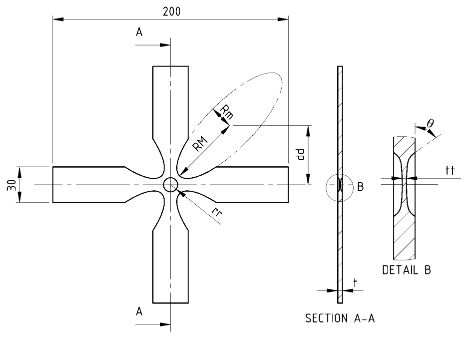

Fig. 1 – Specimen geometry and design variables considered on the optimization process.

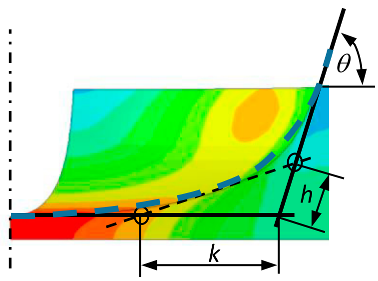

Fig. 2 – Revolved spline used to reduce the specimen center thickness.

3.2. Finite Element Model

The finite element model developed used a symmetric representation of the specimen. With three symmetry planes, only 1/8 of the specimen is modeled with symmetry boundary conditions applied to the three free surfaces, Baptista et al (2015). The model is generated parametrically from the design variables and the mesh is constrained by the geometry. The geometry was divided in to ten separate volumes and inside each volume the number of elements is constant. A total of 32315 hexahedral linear elements and 40548 nodes were used in every mesh. The smallest element used, around a 1 mm radius of the specimen center, had an average dimension of 0.025 mm. Two loading conditions were considered. Both representative of in-plane proportional fatigue lading. The first one is an on-phase loading and the second one has an out-of-phase angle of 180º. Both loads were applied as a distributed load in the arms with a total load of 1 kN. The material behavior was modeled as linear elastic with Poisson coefficient of 0.3. The remain material properties has no influence on the results presented.

3.3. Design Variables

A complete spectrum of specimen dimension was analyzed in this paper. Accordingly, with Fig. 1, the optimization design variables were: 1 - the specimen arms thickness t, this variable was chosen from a list of predefined values, accordingly with the Renard series R10 of preferred numbers (1, 1.2, 1.5, 2, 2.5, 3, 4, 5, 6, 8 and 10 mm), adapted in order to better represent the commercially available sheet metal thickness. 2 - the specimen center reduced thickness tt, this variable allows to increase or decrease the maximum stress level on the specimen center. 3 - the revolved center spline major radius rr, this variable defines the overall size of this feature. 4 - the revolved center spline exit angle θ, this variable allows to control the stress level around the spline. 5 - the parameter k and 6 - the parameter h, which control the spline profile, by changing the position and inclination of the spline tangent line, therefore controlling the uniformity of the central strain distribution. 7 - the major R_m and 8 - minor r_m of the elliptical fillet radius, these parameters control the dimension of this specimen feature. And finally 9 - the elliptical fillet center position dd. A total of 8 variables were optimized for each value of arms thickness t, in order to simplify the problem and to achieve an optimal solution for every value of t.

3.4. Optimization Parameters

Each optimization series was made for a constant value of the arms thickness. The remaining 8 design variables were optimized considering two goal functions: to maximize the stress on the specimen center and to maximize the specimen center reduced thickness tt eq. (2), respectively. The first objective function has the purpose of obtaining the maximum possible stress on the specimen center, optimizing the final specimen geometry. The second objective function has the goal of allowing for several different solutions to be obtained, using the solution Pareto Front, with different tt values. Therefore, for each t value several solutions were obtained with different tt values. As it will be seen in next section, the solutions can be organized in terms of $\mathrm{tt/t}$ or $tt.t$.

$$f_1 = -\sigma_{max};\ f_2 = -tt \tag{2}$$

To accept each solution several conditions were applied as follows: 1 - The average normal strain on a 1 mm radius was accessed in three different directions (loading direction 1, loading direction 2 and on a 45º plane), the maximum strain variation allowed was 2%. 2 - The minimum difference between the center stress level and the remaining specimen (including the inner and outer parts of the elliptical fillet), must be higher than 20% (reasonable limit found by testing experimentally several specimens). 3 - The difference between the maximum and minimum stress level inside the revolved spline must be lower than 10%.

These conditions allowed for optimal strain uniformity and make sure that the maximum stress level always occurs on the specimen center. Therefore the fatigue crack initiation will always begin on the specimen center, and the crack initiation angle is not influenced by the specimen geometry. As these conditions were also verified for both extreme loading conditions, the crack initiation angle is only dependent on the loading path applied.

4. Results

All the results were organized in a first stage by the ratio tt/t (not presented in this present paper). For the optimal solutions found, the range of obtained ratios goes from 0.05 to 0.15. This means that the optimal value of tt stand between 5 and 15 % of the specimen arms thickness. When analyzing the behavior of each design variables, dimensionless by t, it was found that part these ((R_m/t), (r_m/t), (dd/t), (rr/t) and (k/t)) when plotted against t, do not depend on the value of $tt\ /\ t$ and the curves behave in the same way. However this behavior is not as clear for the dimensionless value of the parameter for some variables like (k/t), (θ/t) and the maximum stress at center of specimen

Although this approach is interesting when considering the limitation of the technological processes used to machine the specimens, as a secondary goal, the authors intended to produce a general equation to define as described all the design variables. Therefore a different approach was needed as described below.

4.1. Optimal Specimen

Fig. 3 a) shows the Pareto Front of only a selected series of all the obtained results. As both t and tt varies throughout the optimization process, the value of ($t.tt$) is always increasing, therefore it is a good variable to organize the results as shown in Fig. 3 b) to i), with much better results than the ones obtained by the ratio tt/t.

The end user can therefore define the necessary stress level on the specimen center, in order to induce fatigue crack initiation, Fig. 3 b). The optimal solutions were found for $0.12 \leq t.tt \leq 15$ and the higher stress levels were obtained for a value of $(t.tt) = 0.12$. This value can be achieved with both the arms thickness of 1 or 1.2 mm, and value of the center reduced thickness of 0.10 and 0.12 mm respectively. The maximum stress level obtained for the optimal specimen was 257 MPa/kN (as calculated by the equation provided in Fig. 3 b)). If the end user needs to use higher thickness sheet metal, because of specimen machining constrains, different optimal specimens can be produced, considering higher values of ($t.tt$), and consequently lower maximum stress levels.

As for the remaining design variables, they were also made dimensionless, using the variable $(t \cdot tt)$. Fig. 3 c) to i), which show a clear behavior from all the design variables against ($t.tt$). Once again the value of the spline exit angle ($\theta/t.tt$), have a lower correlation to ($t.tt$), however, at it will be explained in next sub-section, the influence of the parameter θ in the specimen performance is quite insignificant.

With the equations provided in Fig. 3. b) to i) it is possible to design the in-plane biaxial specimen according to the end user needs following next steps:

- Define the maximum stress required during the fatigue tests [MPa/kN];
- Determine the best relation ($t.tt$) by using the equation provided in Fig. 3 b);
- Specify the thickness t [mm] from dimensions available in the market (recommend Renard series as in section 3.3).
- Determine the value of tt, ensuring that it is possible to machine with that size;
- Calculate the remaining design variables by using the equations provided in Fig. 3 c) to i).

4.2. Design Variables Influence

In order to help the end user to fully develop their needed specimen, a sensibility analysis was conducted to all the design variables, to fully understand their influence on the final result. Not only was the maximum stress level on the specimen analyzed, but also the stress and strain uniformity and the stress ratios between the specimen center and the specimen arms was analyzed. Several optimal specimen geometries were considered and the value of each design variable was increased and decreased, using the appropriate increment, individually in order to assess their influence on the specimen performance.

As mentioned, the specimen has two main features represented by the design variables. Starting with the elliptical fillet, the value of the major elliptical fillet radius R_m , is proportional to the maximum stress on the specimen center. Increasing the value of R_m by 0.5 % increases the stress level by 5.5%, but it also reduces the difference between the stress on the specimen center and the specimen arms. By increasing the value of R_m by 0.5 %, the differences on the stress levels decreases 2.5%. There is then the possibility that by increasing the value of R_m the specimen will no longer be acceptable, because the difference between the stress level on the specimen center and on the specimen arms is no longer higher than 20%.

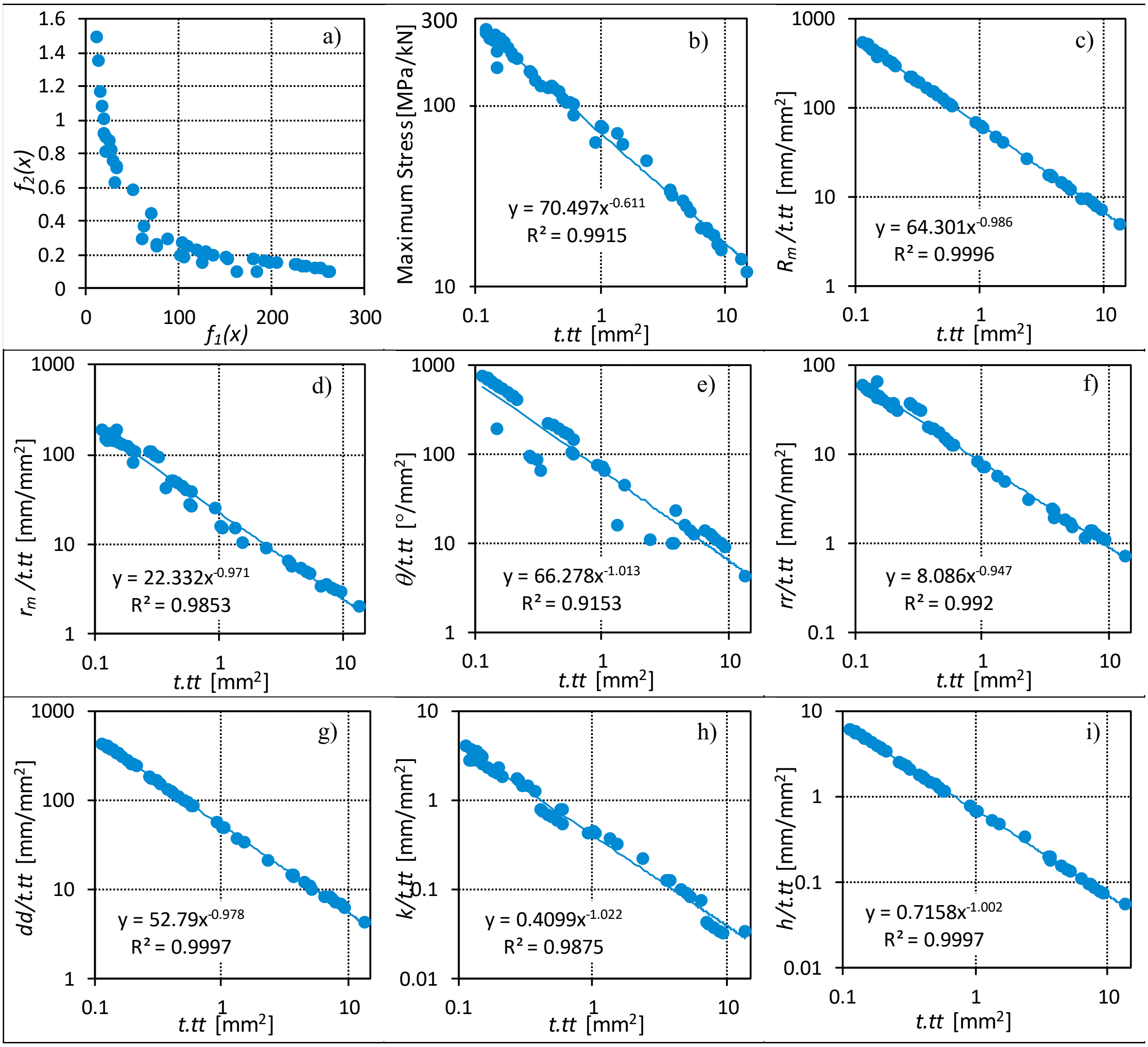

Fig. 3 a) Pareto Fronts obtained for all optimization series; b) Maximum stress level; c) to i) Design variables as a function of $(t \cdot tt)$.

As for the value of the minor elliptical fillet radius r_m, this design variable is proportional to the difference between the stress level on the specimen center and the specimen arms, both on the inner and outer parts. Increasing the value of r_m 4 %, increases these differences by 2%. The influence of r_m on the stress level is negligible.

Finally, the position of the elliptical fillet center dd is inversely proportional to the maximum stress level on the specimen center and directly proportional to the difference between the stress level on the specimen center and the specimen arms. Increasing the value of dd by 2 %, decreases the stress value by 17 % and increases the stress differences by 3.5 %. Therefore it is safe to increase the value of r_m and dd, without jeopardizing the specimen geometry, at the expanse of decreasing the stress level on the specimen center. It is also noteworthy that the elliptical fillet design variables do not affect the strain and stress uniformity on the specimen center, keeping it below the 2 % maximum variation limit.

The second specimen feature that need to be analyzed is the revolved spline used to reduce the specimen center thickness. The exit angle θ has a very insignificant influence on the specimen performance. Increasing the value of θ, increases the stress value on the specimen center, and also increases the strain and stress uniformity. But a 20% variation on the value of θ only produces a 1% increase on the stress level.

The radius of the revolved spline rr is actually one of the most important design variables. It influences all the analyzed parameters. Increasing the value of rr, by 4 %, increases the value of the maximum stress level on the specimen center, by 3.5 %. Unfortunately, it also decreases the difference between the stress level on the specimen center and the specimen arms, by 3.5%. once again, this variable can be used to increase the stress level on the specimen, but it can invalidate the specimen geometry. Finally decreasing the value of rr, decreases the strain and stress uniformity. The justification is very simple, and has been covered by the authors on previous papers, Baptista et al (2015). By increasing the value of rr, the spline becomes less steep on the specimen center increasing the strain and stress uniformity. There is a larger area where the specimen thickness is almost constant. This larger area is also responsible for increasing the value of the stress level on the specimen center. On the other way around, if the value of rr is decreased, the spline inclination becomes steeper, and the strain and stress distributions will be less uniform.

The spline configuration parameters k and h have hardly any influence on the stress level on the specimen center. They should not be used to achieve higher or lower stress levels on the specimen. They should be used to control the strain and stress uniformity on the specimen center. As the authors have previously shown Baptista et al (2015), increasing their values increases the strain and stress uniformity, by smoothing the spline inclination and by increasing the uniform area around the specimen center. They do have a negative effect, by increasing the value of k by 14% or the value of h by 7%, the stress difference between the specimen center and the specimen arms decreases by 2.5%. As mentioned before this can in fact invalidate the specimen geometry.

Finally, the main design variable in the center reduced thickness tt value. This design variable also affects all the analyzed parameters. Increasing the value of tt by 20 %, decreases the value of the maximum stress level by 10 %. While decreasing the value of tt by 22 %, increases the stress level by 14 %. Increasing the value of tt, by 20 %, also decreases the stress differences between the specimen center and the specimen arms by 5 %. And it is also responsible for increasing the strain and stress uniformity on the specimen center. As mentioned above, increasing the center reduced thickness, makes the spline more uniform around the specimen center.

Using this information the end user can design the final specimen as necessary. As a final remark the end user should keep in mind that by increasing the stress level on the specimen, it normally is reducing the stress differences between the specimen center and specimen arms. Also by controlling the spline profile it is possible to achieve higher strain and stress distribution uniformities, but this actually leads to a geometry where the spline is very horizontal at the specimen center, and ends with a high exit angle.

5. Conclusions

As final remarks of this work, and based on previous experience the authors can conclude that:

- The Direct Multi-Search method was able to optimize complex problems, with eight design variables and two objective functions, for each one of the nominal arms thickness defined;
- The Pareto Front obtained for each material thickness, defined using the Renard series of preferred number, allow the end user to choose the optimal specimen geometry as intended;
- One easy way to organize the results is to classify the solution as a function of the ratio between the specimen arms thickness and the specimen center reduced thickness;
- Most the design variables correlate well with the arms thickness of the specimen, and are not depended on the value of the ratio between the specimen arms thickness and the specimen center reduced thickness;
- A second and more efficient way to organize the results is to calculate the parameter $t.tt$, as the optimal solutions can be organized by its increasing value;
- All the design variables were correlated to this new parameter and mathematical expressions that define the design variables were obtained;

- The end user can therefore choose the optimal solution from the expressions presented to generate the necessary solution;
- The influence of all the design variables was assessed, to give the end user, the necessary knowledge to change and adapt the present solutions as needed;
- The features used to design the current specimen allowed for optimal solution with high but uniform stress distributions on the specimen center.

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